Radiative Generation of θ_{13} with the Seesaw Threshold Effect

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Abstract

We examine whether an appreciable value of the lepton flavor mixing angle θ_{13} at the electroweak scale $\Lambda_{\rm EW}$ can be radiatively generated from $\theta_{13} = 0^\circ$ at the GUT scale $\Lambda_{\rm GUT}$. It is found that the renormalization-group running and seesaw threshold effects may lead to $\theta_{13} \sim 5^\circ$ at low energies for two simple large-maximal mixing patterns of the MNS matrix in the minimal supersymmetric standard model. If θ_{12} is sufficiently large at $\Lambda_{\rm GUT}$, it will be possible to radiatively produce $\theta_{13} \sim 5^\circ$ at $\Lambda_{\rm EW}$ both in the standard model and in its supersymmetric extensions. The mass spectrum of three heavy right-handed Majorana neutrinos and the cosmological baryon number asymmetry via leptogenesis are also calculated.

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I. INTRODUCTION

The recent solar [1], atmospheric [2], reactor (KamLAND [3] and CHOOZ [4]) and accelerator (K2K [5]) neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive particles and their mixing involves two large angles ($\theta_{12} \sim 33^\circ$ and $\theta_{23} \sim 45^\circ$) and one small angle ($\theta_{13} < 13^\circ$). How small θ_{13} is remains an open question, but a global analysis of the presently available neutrino oscillation data [6] indicates that θ_{13} is most likely to lie in the range $4^\circ \leq \theta_{13} \leq 6^\circ$. One important target of the future neutrino experiments is just to measure θ_{13} [7].

The smallness of θ_{13} requires a good theoretical reason, which might simultaneously account for the largeness of θ_{12} and θ_{23}. If $\theta_{13} = 0^\circ$ held, there should exist a kind of new flavor symmetry which forbids flavor mixing between the first and third lepton families. While such a new symmetry is unlikely to exist at or below the electroweak scale ($\Lambda_{\rm EW} \sim 10^2$ GeV), it might show up at a superhigh scale – e.g., the scale of grand unified theories ($\Lambda_{\rm GUT} \sim 10^{16}$ GeV). Then a natural way to break this flavor symmetry and obtain $\theta_{13} \neq 0^\circ$ in a specific model is to run relevant parameters of the model from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$ by making use of the renormalization group equations (RGEs) [8] and taking account of the seesaw threshold effects [9], either in the standard model (SM) or in the minimal supersymmetric standard model (MSSM). Antusch *et al* have recently presented two simple examples (one with $\theta_{12} = \theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ at $\Lambda_{\rm GUT}$ [10], and the other with $\theta_{12} = \theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$ at $\Lambda_{\rm GUT}$ [11]) to radiatively generate $\theta_{12} \sim 33^\circ$ and $\theta_{13} \neq 0^\circ$, but their primary interest is in θ_{12} and their results for θ_{13} are far below the best-fit values of θ_{13} obtained from the global analysis [6]. Although the low-scale output of θ_{13} is somehow adjustable by scanning the parameter space of a given model at the GUT scale, we find that it is highly nontrivial to obtain $\theta_{13}(\Lambda_{\rm EW}) \sim 5^\circ$ from $\theta_{13}(\Lambda_{\rm GUT}) = 0^\circ$ and fit all experimental data of neutrino oscillations in the meantime.

The main purpose of this paper is to examine whether an appreciable magnitude of θ_{13} can be radiatively generated, from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$, through the seesaw thresholds. We shall consider four instructive patterns of the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix $V_{\rm MNS}$ at $\Lambda_{\rm GUT}$ as typical examples:

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\begin{aligned}
&\text{Pattern (A)}: \qquad V_{\rm MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} P_\delta \, , \\
&\text{Pattern (B)}: \qquad V_{\rm MNS} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} P_\delta \, , \\
&\text{Pattern (C)}: \qquad V_{\rm MNS} = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix} P_\delta \, , \\
&\text{Pattern (D)}: \qquad V_{\rm MNS} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \end{pmatrix} P_\delta \, ,
\end{aligned} \tag{1}
$$

where $P_\delta \equiv \mathrm{Diag}\{1, 1, e^{i\delta}\}$ with δ being a CP-violating phase in the standard parametrization of $V_{\rm MNS}$ [12] [1]. While patterns (A) and (B) with CP conservation (i.e., $\delta = 0^\circ$) have been discussed in Refs. [10] and [11], we shall show that δ can actually play an important role in the radiative generation of θ_{13}. Patterns (C) [13] and (D) [14] are phenomenologically favored to account for current experimental data of solar and atmospheric neutrino oscillations. We find that the RGE running and seesaw threshold effects may allow us to obtain $\theta_{13} \sim 5^\circ$ at low energies from both patterns (C) and (D) in the MSSM. If $\theta_{12} \sim 60^\circ$ is taken at $\Lambda_{\rm GUT}$, it will be possible to radiatively produce $\theta_{13} \sim 5^\circ$ at $\Lambda_{\rm EW}$ both in the SM and in the MSSM. As a by-product, the mass spectrum of three heavy Majorana neutrinos and the cosmological baryon number asymmetry via leptogenesis are also calculated.

II. RGE RUNNING AND THRESHOLD EFFECTS

Let us make a simple modification of the SM by introducing three heavy right-handed neutrinos N_i (for $i = 1, 2, 3$) and keeping the Lagrangian of electroweak interactions invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge transformation. In this case, the Lagrangian relevant for lepton masses can be written as

$$-\mathcal{L}_{\rm lepton} \; = \; \bar{l}_{\rm L} Y_l e_{\rm R} H + \bar{l}_{\rm L} Y_\nu \nu_{\rm R} H^{\rm c} + \frac{1}{2} \overline{\nu^{\rm c}_{\rm R}} M_{\rm R} \nu_{\rm R} + {\rm h.c.} \; , \tag{2}$$

where $l_{\rm L}$ denotes the left-handed lepton doublet; $e_{\rm R}$ and $\nu_{\rm R}$ stand respectively for the right-handed charged lepton and Majorana neutrino singlets; and H is the Higgs-boson weak isodoublet (with $H^{\rm c} \equiv i\sigma_2 H^*$). If the MSSM is taken into account, one may similarly write out the Lagrangian relevant for lepton masses:

$$-\mathcal{L}'_{\rm lepton} \; = \; \bar{l}_{\rm L} Y_l e_{\rm R} H_1 + \bar{l}_{\rm L} Y_\nu \nu_{\rm R} H_2 + \frac{1}{2} \overline{\nu^{\rm c}_{\rm R}} M_{\rm R} \nu_{\rm R} + {\rm h.c.} \; , \tag{3}$$

where H_1 and H_2 (with hypercharges $\pm 1/2$) are the MSSM Higgs doublets. To obtain the effective (left-handed) neutrino mass matrix, a common approach is to integrate $M_{\rm R}$ out of the full theory. This corresponds to a replacement of the last two terms in $\mathcal{L}_{\rm lepton}$ or $\mathcal{L}'_{\rm lepton}$ by a dimension-5 operator, whose coupling matrix takes the well-known seesaw form $\kappa = -Y_\nu M_{\rm R}^{-1} Y_\nu^T$ [15]. However, the threshold effects in such a naive treatment have to be taken into account, because the mass eigenvalues of N_i may have a strong hierarchy (for example, $M_3 \gg M_2 \gg M_1$).

We take $\mathcal{L}_{\rm lepton}$ or $\mathcal{L}'_{\rm lepton}$ for granted at the GUT scale, where the Yukawa interactions of quarks and Higgs bosons with the coupling matrices $Y_{\rm u}$ (up) and $Y_{\rm d}$ (down) can similarly be written out. To evolve the lepton mixing parameters from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$ in a generic seesaw model [2], one has to make use of a series of effective theories which are obtained by

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[1] For simplicity and illustration, we only take account of a single CP-violating phase in this work.

[2] We assume the supersymmetry breaking scale $\Lambda_{\rm SUSY}$ to be close to the electroweak scale $\Lambda_{\rm EW}$, just for the sake of simplicity. Even if $\Lambda_{\rm SUSY}/\Lambda_{\rm EW} \sim 10$ holds, the relevant RGE running effects between these two scales are negligibly small for the physics under consideration.

integrating out the heavy right-handed singlets N_i step by step at their mass thresholds. The derivation of the one-loop RGEs and the method for dealing with the effective theories have been presented in Ref. [9] in an elegant way. Here we summarize a few essential steps to be taken in treating the seesaw threshold effects.

(a) We use the one-loop RGEs to run Y_ν and $M_{\rm R}$ from $\Lambda_{\rm GUT}$ to the heaviest right-handed neutrino mass scale M_3. A proper unitary transformation of the right-handed neutrino fields allows us to diagonalize $M_{\rm R}$ at M_3 – namely, $U^\dagger_{\rm R} M_{\rm R} U^*_{\rm R} = {\rm Diag}\{M_1, M_2, M_3\}$. Then Y_ν is transformed into $Y_\nu U^*_{\rm R}$. The effective neutrino coupling matrix $\kappa_{(3)}$ can be obtained by integrating out M_3. In this case, we denote [3]

$$Y_{\nu(3)} \; = \; Y_\nu U^*_{\rm R} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \; , \qquad \hat{Y}_{\nu(3)} \; = \; Y_\nu U^*_{\rm R} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \; , \qquad M_{\rm R(3)} \; = \; \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \tag{4}$$

and get the tree-level matching relation $\kappa_{(3)} = -\hat{Y}_{\nu(3)} M_3^{-1} \hat{Y}^T_{\nu(3)}$, where all variables have been set to the scale $\mu = M_3$.

(b) We further run $Y_{\nu(3)}$, $M_{\rm R(3)}$ and $\kappa_{(3)}$ from M_3 down to the intermediate right-handed neutrino mass scale M_2. Because the RGE running effect may spoil the diagonal feature of $M_{\rm R(3)}$, a re-diagonalization of $M_{\rm R(3)}$ at M_2 should be done by means of a 2×2 unitary transformation matrix $\tilde{U}_{\rm R}$. Integrating out M_2, we arrive at

$$Y_{\nu(2)} \; = \; Y_{\nu(3)} \tilde{U}^*_{\rm R} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; , \qquad \hat{Y}_{\nu(2)} \; = \; Y_{\nu(3)} \tilde{U}^*_{\rm R} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; , \qquad M_{\rm R(2)} \; = \; \left(\, M_1 \, \right) \tag{5}$$

and the tree-level matching condition $\kappa_{(2)} = \kappa_{(3)} - \hat{Y}_{\nu(2)} M_2^{-1} \hat{Y}^T_{\nu(2)}$, where all variables have been set to the scale $\mu = M_2$.

(c) We follow a similar way to run $Y_{\nu(2)}$, $M_{\rm R(2)}$ and $\kappa_{(2)}$ from M_2 down to the lightest right-handed neutrino mass scale M_1. As $M_{\rm R(2)}$ is actually a 1×1 mass matrix, it does not need to be re-diagonalized at M_1. Integrating out M_1, we obtain

$$\kappa \; \equiv \; \kappa_{(1)} \; = \; \kappa_{(2)} - \hat{Y}_{\nu(1)} M_1^{-1} \hat{Y}^T_{\nu(1)} \; , \tag{6}$$

where $\hat{Y}_{\nu(1)} = Y_{\nu(2)}$ holds, and all variables have been set to the scale $\mu = M_1$.

(d) Finally, we run κ from M_1 down to the electroweak scale $\Lambda_{\rm EW}$. The one-loop RGE governing the evolution of κ is given by [8]

$$16\pi^2 \frac{{\rm d}\kappa}{{\rm d}t} \; = \; \alpha\kappa + C \left[\left(Y_l Y_l^\dagger \right) \kappa + \kappa \left(Y_l Y_l^\dagger \right)^T \right] \; , \tag{7}$$

where $t = \ln(\mu/M_1)$ with μ being the renormalization scale. We have $C = -1.5$, $\alpha \approx -3g_2^2 + 6f_t^2 + \lambda$ in the SM and $C = 1$, $\alpha \approx -1.2g_1^2 - 6g_2^2 + 6f_t^2$ in the MSSM [16], where $g_{1,2}$ denote the gauge couplings, f_t denotes the top-quark Yukawa coupling, and λ denotes

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[3] It should be noted that our notations (in particular, $\kappa = -Y_\nu M_{\rm R}^{-1} Y_\nu^T$) are somehow different from those of Ref. [9], where the seesaw formula $\kappa = 2Y_\nu^T M_{\rm R}^{-1} Y_\nu$ has been adopted.

the Higgs self-coupling in the SM. After spontaneous gauge symmetry breaking, we arrive at the fermion mass matrices $M_\nu = v^2\kappa$, $M_l = vY_l$, $M_{\rm u} = vY_{\rm u}$ and $M_{\rm d} = vY_{\rm d}$ in the SM; and $M_\nu = v^2\kappa\sin^2\beta$, $M_l = vY_l\cos\beta$, $M_{\rm u} = vY_{\rm u}\sin\beta$ and $M_{\rm d} = vY_{\rm d}\cos\beta$ in the MSSM, where $v \approx 174$ GeV stands for the vacuum expectation value of the neutral Higgs field in the SM, and $\tan\beta$ represents the ratio of two vacuum expectation values in the MSSM.

III. INITIAL CONDITIONS AND ASSUMPTIONS

The lepton (or quark) flavor mixing matrix $V_{\rm MNS}$ (or $V_{\rm CKM}$) arises from the mismatch between the diagonalizations of Y_l (or $Y_{\rm u}$) and κ (or $Y_{\rm d}$). Without loss of generality, we arrange Y_l and $Y_{\rm u}$ to be diagonal, real and positive at $\Lambda_{\rm GUT}$; i.e.,

$$Y_{\rm u} \;=\; \frac{1}{\Omega_1}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} , \qquad\qquad Y_l \;=\; \frac{1}{\Omega_2}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} , \tag{8}$$

where $\Omega_1 = \Omega_2 = v$ in the SM, and $\Omega_1 = v\sin\beta$ and $\Omega_2 = v\cos\beta$ in the MSSM. In this flavor basis, $Y_{\rm d}$ and Y_ν can generally be expressed as

$$Y_{\rm d} \;=\; \frac{1}{\Omega_2}V_{\rm CKM}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} U_{\rm d} \; , \qquad Y_\nu \;=\; y_\nu V_\nu \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} U_\nu \; , \tag{9}$$

where r_1, r_2 and y_ν are three real and positive dimensionless parameters characterizing the eigenvalues of Y_ν; and V_ν, U_ν and $U_{\rm d}$ are three 3×3 unitary matrices. It is obvious that the complex phases of V_ν and U_ν can be re-arranged, such that the former contains a single irremovable CP-violating phase as $V_{\rm CKM}$ does. Furthermore, one may redefine the relevant right-handed fields of quarks and neutrinos to rotate away $U_{\rm d}$ from $Y_{\rm d}$ and U_ν from Y_ν. Such a transformation of Y_ν is equivalent to absorbing U_ν into the Majorana mass matrix $M_{\rm R}$, which is not required to be diagonal at $\Lambda_{\rm GUT}$. Once $U_{\rm d}$ and U_ν are rejected, we are only left with seven unknown parameters in the above Yukawa coupling matrices (namely, three eigenvalues of Y_ν and four mixing parameters of V_ν).

To fix the pattern of $M_{\rm R}$ at $\Lambda_{\rm GUT}$, we extrapolate the effective neutrino coupling matrix κ and the lepton flavor mixing matrix $V_{\rm MNS}$ up to the GUT scale:

$$\kappa \;=\; \frac{1}{\Omega_1^2}V_{\rm MNS}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T_{\rm MNS} \; , \tag{10}$$

where m_i (for $i = 1,2,3$) denote the physical masses of three light neutrinos. Then $M_{\rm R}$ can be determined from the inverted seesaw formula $M_{\rm R} = -Y_\nu^T\kappa^{-1}Y_\nu$ [19]. Note that $V_{\rm MNS}$ consists of three mixing angles and three CP-violating phases. For the sake of simplicity, here we only take account of the "Dirac-like" phase of $V_{\rm MNS}$. Then the unitary matrices $V_{\rm CKM}$, $V_{\rm MNS}$ and V_ν may universally be parametrized as

$$V \;=\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{11}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \theta_{ij}$. In the limit of $\theta_{13} = 0°$, the complex phase of $V_{\rm MNS}$ actually serves as a "Majorana-like" phase and may significantly affect the RGE running behaviors of neutrino masses and lepton flavor mixing angles [17]. Four typical patterns of $V_{\rm MNS}$, as already listed in Eq. (1), will be taken in our following investigation.

To be specific, we only pay attention to the normal mass hierarchy of three light neutrinos (i.e., $m_3 > m_2 > m_1$). Then we arrive at $m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm sun}}$ and $m_3 = \sqrt{m_2^2 + \Delta m^2_{\rm atm}}$, where $\Delta m^2_{\rm sun}$ and $\Delta m^2_{\rm atm}$ denote the mass-squared differences of solar and atmospheric neutrino oscillations. Since the low-scale values of (m_u, m_c, m_t), (m_d, m_s, m_b), (m_e, m_μ, m_τ) and $(\Delta m^2_{\rm sun}, \Delta m^2_{\rm atm})$ are all known, we just have a single unknown mass parameter (m_1). On the other hand, four parameters of $V_{\rm CKM}$ are also known at low energies [18]. Given a special pattern of $V_{\rm MNS}$ in Eq. (1), only its CP-violating phase is not fixed. In short, we are totally left with nine free parameters at $\Lambda_{\rm GUT}$: the lightest neutrino mass m_1, three eigenvalues of Y_ν, four mixing parameters of V_ν and the CP-violating phase of $V_{\rm MNS}$. We should also specify the value of the Higgs mass m_H in the SM and that of $\tan\beta$ in the MSSM when numerically solving the relevant RGEs.

Although y_ν, r_1 and r_2 are arbitrary parameters, they are expected to be of or below $\mathcal{O}(1)$. The unknown rotation and phase angles of V_ν and $V_{\rm MNS}$ are allowed to take possible values in the range between 0 and 2π. As for those parameters whose sizes are known at low energies, one may properly adjust their initial values at $\Lambda_{\rm GUT}$ and run the RGEs to reproduce their low-scale values within reasonable error bars. We are then able to fix the parameter space by fitting all relevant low-scale data. Such a phenomenological approach will allow us to examine whether $\theta_{13}(\Lambda_{\rm EW}) \sim 5°$ can be generated from $\theta_{13}(\Lambda_{\rm GUT}) = 0°$ for $V_{\rm MNS}$. Naively, we speculate that an appreciable RGE enhancement of θ_{13} may take place either between the scales $\Lambda_{\rm GUT}$ and M_1 or between the scales M_1 and $\Lambda_{\rm EW}$. In either case, the masses of three light neutrinos are required to be nearly degenerate. As the seesaw threshold effect can significantly affect the RGE running behaviors in most cases [9–11,20], it is more likely that the dominant RGE enhancement of θ_{13} occurs between $\Lambda_{\rm GUT}$ and M_1. We shall demonstrate this observation in our subsequent numerical calculations.

IV. NUMERICAL EXAMPLES AND DISCUSSIONS

We carry out a numerical analysis of the RGE running and seesaw threshold effects by following the strategies outlined above and taking four typical patterns of $V_{\rm MNS}$ at the GUT scale. Our results are summarized in Table 1 and Figs. 1 and 2. Some comments and discussions are in order.

(1) We have carefully examined the sensitivity of $V_{\rm MNS}(\Lambda_{\rm EW})$ to the value of every free parameter at $\Lambda_{\rm GUT}$. We find that the inputs of m_1, y_ν, $\theta_{12}^{\rm MNS}$, $\delta_{\rm MNS}$, θ_{13} of V_ν and $\Delta m^2_{\rm sun}$ appear to be very important in adjusting the output of $\theta_{13}^{\rm MNS}$ at $\Lambda_{\rm EW}$, while the other parameters mainly play a role in fine-tuning the results. In particular, the effects of r_1, r_2, θ_{12} of V_ν, θ_{23} of V_ν and δ of V_ν are insignificant, because the dominant RGE enhancement of $\theta_{13}^{\rm MNS}$ takes place above the heaviest right-handed neutrino mass scale M_3. Hence we have simply fixed $\theta_{12} = \theta_{23} = \delta = 0°$ for V_ν at $\Lambda_{\rm GUT}$. Note that the initial values of $\delta_{\rm MNS}$ and θ_{13} of V_ν are important in specifying the running tendency of two neutrino mass-squared differences and three mixing angles of $V_{\rm MNS}$, and they have to be sufficiently large (as shown

in Table 1) in order to generate an appreciable magnitude of $\theta_{13}^{\rm MNS}$ at low energies. To be more explicit, the evolution of $\theta_{12}^{\rm MNS}$ strongly relies on $\delta_{\rm MNS}$ in the fitting, while the enhancement of $\theta_{13}^{\rm MNS}$ (from zero at $\Lambda_{\rm GUT}$ to a few degrees at $\Lambda_{\rm EW}$) requires a big input value for θ_{13} of V_ν ($\sim 45^\circ$, for example). It is also worth remarking that the parameter space obtained here should by no means be unique; or rather, it mainly serves for illustration. To exhaustively explore the allowed ranges of all relevant parameters is a quite lengthy work and will be presented elsewhere [21].

(2) Restricting ourselves to the typical parameter space illustrated in Table 1, we find that it is difficult to produce $\theta_{13}^{\rm MNS} \sim 5^\circ$ at $\Lambda_{\rm EW}$ in the SM. Our results yield $\theta_{13}^{\rm MNS} \sim 3^\circ$ for pattern (A) and $\theta_{13}^{\rm MNS} \sim 1.5^\circ$ for patterns (B), (C) and (D) at low energies. In the MSSM, however, a more strong RGE enhancement of $\theta_{13}^{\rm MNS}$ becomes possible. For example, we obtain $\theta_{13}^{\rm MNS} \sim 3^\circ$ for pattern (B) with $\tan\beta \sim 10$ and $\theta_{13}^{\rm MNS} \sim 5^\circ$ for patterns (C) and (D) with $\tan\beta \sim 20$ at the electroweak scale. We think that the numerical results in the supersymmetric case are encouraging for model building, because a new kind of flavor symmetry at $\Lambda_{\rm GUT}$ might naturally lead to the bi-maximal mixing pattern (B) or the large-maximal mixing patterns (C) and (D).

(3) The running behavior of $\theta_{12}^{\rm MNS}$ deserves some more remarks. As already noticed in Refs. [8–11], the evolution of $\theta_{12}^{\rm MNS}$ from $\Lambda_{\rm EW}$ to $\Lambda_{\rm GUT}$ (or vice versa) depends sensitively on how big its initial value is and whether three light neutrinos have a near mass degeneracy. Figs. 1 and 2 illustrate that $\theta_{12}^{\rm MNS}(\Lambda_{\rm EW}) \sim 33^\circ$ can radiatively be obtained, either from $\theta_{12}^{\rm MNS}(\Lambda_{\rm GUT}) = 0^\circ$ in pattern (A) or from $\theta_{12}^{\rm MNS}(\Lambda_{\rm GUT}) \sim (30^\circ - 45^\circ)$ in patterns (B), (C) and (D). For pattern (A) or (B), the RGE running of $\theta_{12}^{\rm MNS}$ in the MSSM is quite similar to that in the SM. The reason for this similarity is simply that the magnitudes of m_1 (SM) and m_1 (MSSM) at $\Lambda_{\rm EW}$ are comparable (~ 0.05 eV) and the value of $\tan\beta$ in the MSSM case is mild (~ 10). When patterns (C) and (D) are concerned, however, the running behavior of $\theta_{12}^{\rm MNS}$ in the MSSM is more violent than that in the SM. The reason for such a remarkable difference is two-fold: first, $m_1(\Lambda_{\rm EW}) \sim 0.14$ eV and $m_1(\Lambda_{\rm GUT}) \sim 0.20$ eV in the MSSM case imply that the masses of three light neutrinos are nearly degenerate in the entire running course – this near mass degeneracy can significantly affect the behavior of $\theta_{12}^{\rm MNS}$ [8]; second, the large value of $\tan\beta$ (~ 20) plays an important role in enhancing the RGE evolution of light neutrino masses and flavor mixing angles (e.g., $\dot{\theta}_{12}^{\rm MNS} \propto (1+\tan^2\beta)$ [8]).

(4) We stress that $\theta_{13}^{\rm MNS} \sim 5^\circ$ at $\Lambda_{\rm EW}$ can be radiatively generated from $\theta_{13}^{\rm MNS} = 0^\circ$ at $\Lambda_{\rm GUT}$ even in the SM, only if we go beyond the four simple patterns considered above. A key point is to assume $\theta_{12}^{\rm MNS}$ to be large enough at the GUT scale. Such an example is presented in Fig. 3, where $\theta_{12}^{\rm MNS} = 67^\circ$ (SM) versus $\theta_{12}^{\rm MNS} = 63^\circ$ (MSSM) has been taken. One may see that the dominant RGE suppression of $\theta_{12}^{\rm MNS}$ occurs from $\Lambda_{\rm GUT}$ to M_3, and the dominant RGE enhancement of $\theta_{13}^{\rm MNS}$ takes place in the same region. Why $\theta_{12}^{\rm MNS} > 45^\circ$ holds at $\Lambda_{\rm GUT}$ is certainly a big question. Putting aside this question, we remark that $\theta_{12}^{\rm MNS} \sim 33^\circ$ at low energies can in principle be produced from an arbitrary value of $\theta_{12}^{\rm MNS}$ at the GUT scale via the seesaw threshold effects. Furthermore, the radiative generation of $\theta_{13}^{\rm MNS}$ is highly sensitive to the initial condition of $\theta_{12}^{\rm MNS}$. We observe that the running of $\theta_{23}^{\rm MNS}$ is rather stable, unlike $\theta_{12}^{\rm MNS}$ and $\theta_{13}^{\rm MNS}$. Note that the CP-violating phase $\delta_{\rm MNS}$ is also stable against radiative corrections from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$. This conclusion is true for both the example under consideration and the four patterns discussed above.

(5) A by-product of our analysis is the determination of three heavy right-handed neu-

trino masses, as shown in Table 1. We see that they have a clear normal hierarchy. It is then possible to calculate the cosmological baryon number asymmetry via leptogenesis [22]. Instead of describing the technical details of leptogenesis [23], we only give a brief summary of its essential points in the following. Lepton number violation induced by the third term of $\mathcal{L}_{\rm lepton}$ in Eq. (1) or of $\mathcal{L}'_{\rm lepton}$ in Eq. (2) allows decays of three heavy Majorana neutrinos N_i to happen: $N_i \rightarrow l + h$ and $N_i \rightarrow \bar{l} + h^{\rm c}$, where $h = H$ in the SM or $h = H_2^{\rm c}$ in the MSSM. Because each decay mode occurs at both tree and one-loop levels (via the self-energy and vertex corrections), the interference between these two decay amplitudes may result in a CP-violating asymmetry ε_i between $N_i \rightarrow l + h$ and its CP-conjugated process. If the interactions of N_1 are in thermal equilibrium when N_3 and N_2 decay, the asymmetries ε_3 and ε_2 can be erased before N_1 decays. Then only the asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives. This CP-violating asymmetry may lead to a net lepton number asymmetry $Y_{\rm L} \propto \varepsilon_1$, and the latter is eventually converted into a net baryon number asymmetry $Y_{\rm B}$ via the nonperturbative sphaleron processes [24]: $Y_{\rm B} \approx -0.55 Y_{\rm L}$ in the SM or $Y_{\rm B} \approx -0.53 Y_{\rm L}$ in the MSSM. We follow these steps to evaluate $Y_{\rm B}$ for four patterns listed in Table 1. It turns out that only pattern (B) can yield an appreciable result, which lies in the generous range $0.7 \times 10^{-10} \lesssim Y_{\rm B} \lesssim 1.0 \times 10^{-10}$ drawn from the recent WMAP observational data [25]. Compared with pattern (B), patterns (A), (C) and (D) have relatively small values of M_1 ($\sim 10^9$ GeV) and M_1/M_2 ($\sim 3 \times 10^{-2}$). The outputs of $Y_{\rm B}$ in these three patterns are therefore suppressed and below the observational result. If the example shown in Fig. 3 is taken into account, we may obtain $Y_{\rm B} \approx 7.7 \times 10^{-11}$ (SM) or $Y_{\rm B} \approx 8.0 \times 10^{-11}$ (MSSM), which is compatible with the WMAP data.

Finally, it is worthwhile to point out that we have also taken a look at the "democratic" neutrino mixing pattern [26]

$$V_{\rm MNS} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix} \tag{12}$$

at the GUT scale and examined the possibility to radiatively produce $\theta_{13} \sim 5^\circ$ at the electroweak scale and to simultaneously fit all experimental data of neutrino oscillations. We find that it is extremely difficult, if not impossible, to obtain $\theta_{23} \sim 45^\circ$ at low energies. The reason is simply that the initial value of θ_{23} ($\approx 54.7^\circ$) is not close to 45° and the RGE evolution cannot significantly change the magnitude of this mixing angle. Therefore, we argue that radiative corrections might not be a very natural way to break the lepton flavor democracy. The latter could be explicitly broken in some other possible ways at or below the GUT scale.

V. SUMMARY

We have conjectured that the lepton flavor mixing angle θ_{13} might be vanishing at the GUT scale due to the existence of a kind of new flavor symmetry. Starting from this point of view, we have examined whether an appreciable value of θ_{13} at low energies can be radiatively generated via the RGE running and seesaw threshold effects. Four simple but typical patterns of the lepton flavor mixing matrices have been considered as the initial conditions

at $\Lambda_{\rm GUT}$. It is found that the dominant RGE enhancement of θ_{13} takes place from $\Lambda_{\rm GUT}$ to the heaviest right-handed neutrino mass scale. For two large-maximal mixing patterns, $\theta_{13}(\Lambda_{\rm EW}) \sim 5^\circ$ can be radiatively produced in the MSSM. We have also demonstrated that it is possible to obtain $\theta_{13} \sim 5^\circ$ at $\Lambda_{\rm EW}$ from $\theta_{13} = 0^\circ$ at $\Lambda_{\rm GUT}$ in the SM, if the initial value of θ_{12} is large enough. As a useful by-product, the mass spectrum of three heavy Majorana neutrinos is determined and the cosmological baryon number asymmetry via leptogenesis is calculated.

Although the numerical examples presented in this paper are mainly for the purpose of illustration, they are quite suggestive for model building. Of course, only the future neutrino experiments can tell us how small θ_{13} is. But we believe that the radiative generation of θ_{13} from high to low energies is an interesting theoretical approach towards understanding the smallness of θ_{13}, and it might indicate some useful hints about the underlying flavor symmetry which is associated with the dynamics of lepton flavor mixing and CP violation.

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TABLES

TABLE I. Numerical examples for radiative generation of θ_{13} via the RGE evolution and seesaw threshold effects. The cosmological baryon number asymmetry $Y_{\rm B}$ is also computed. Note that we have taken $\theta_{12} = \theta_{23} = \delta = 0°$ for V_ν at $\Lambda_{\rm GUT}$.

Inputs ($\Lambda_{\rm GUT}$)	Pattern (A)	Pattern (B)	Pattern (C)	Pattern (D)	Model
m_1 (eV)	0.12	0.08	0.06	0.06	SM
	0.08	0.10	0.20	0.20	MSSM
$\Delta m^2_{\rm sun}$ (eV^2)	1.0×10^{-4}	1.8×10^{-4}	1.6×10^{-4}	1.2×10^{-4}	SM
	2.1×10^{-4}	2.6×10^{-4}	5.8×10^{-4}	6.1×10^{-4}	MSSM
$\Delta m^2_{\rm atm}$ (eV^2)	7.9×10^{-3}	7.6×10^{-3}	8.0×10^{-3}	7.8×10^{-3}	SM
	5.9×10^{-3}	5.7×10^{-3}	5.1×10^{-3}	5.1×10^{-3}	MSSM
δ of $V_{\rm MNS}$	70°	90°	75°	45°	SM
	70°	100°	115°	115°	MSSM
θ_{13} of V_ν	35°	50°	45°	45°	SM
	27°	39°	45°	45°	MSSM
y_ν	0.9	0.8	0.9	0.9	SM
	0.9	0.8	0.4	0.4	MSSM
r_1	1/300	1/43	1/433	1/433	SM
	1/300	1/43	1/433	1/433	MSSM
r_2	1/19	1/15	1/19	1/19	SM
	1/19	1/15	1/19	1/19	MSSM
m_H (GeV)	120	120	120	120	SM
$\tan\beta$	10	10	19	21	MSSM
Outputs ($\Lambda_{\rm EW}$)	**Pattern (A)**	**Pattern (B)**	**Pattern (C)**	**Pattern (D)**	**Model**
m_1 (eV)	0.069	0.046	0.034	0.034	SM
	0.053	0.067	0.14	0.14	MSSM
$\Delta m^2_{\rm sun}$ (eV^2)	6.9×10^{-5}	6.9×10^{-5}	6.9×10^{-5}	7.0×10^{-5}	SM
	6.8×10^{-5}	7.3×10^{-5}	7.2×10^{-5}	6.9×10^{-5}	MSSM
$\Delta m^2_{\rm atm}$ (eV^2)	2.6×10^{-3}	2.6×10^{-3}	2.7×10^{-3}	2.6×10^{-3}	SM
	2.6×10^{-3}	2.6×10^{-3}	2.6×10^{-3}	2.6×10^{-3}	MSSM
θ_{13} of $V_{\rm MNS}$	3.1°	1.6°	1.5°	1.4°	SM
	2.2°	3.3°	4.7°	4.6°	MSSM
Outputs (M_1)	**Pattern (A)**	**Pattern (B)**	**Pattern (C)**	**Pattern (D)**	**Model**
M_1 (GeV)	3.0×10^{9}	1.8×10^{11}	2.9×10^{9}	2.2×10^{9}	SM
	3.9×10^{9}	1.4×10^{11}	2.1×10^{8}	2.0×10^{8}	MSSM
M_2 (GeV)	8.4×10^{11}	9.9×10^{11}	1.0×10^{12}	8.9×10^{11}	SM
	1.6×10^{12}	1.4×10^{12}	6.9×10^{10}	6.5×10^{10}	MSSM
M_3 (GeV)	7.9×10^{13}	1.2×10^{14}	1.7×10^{14}	2.7×10^{14}	SM
	8.8×10^{13}	6.5×10^{13}	1.5×10^{13}	1.4×10^{13}	MSSM
$Y_{\rm B}$	1.9×10^{-12}	8.9×10^{-11}	1.7×10^{-12}	6.1×10^{-13}	SM
	1.7×10^{-12}	7.5×10^{-11}	1.6×10^{-13}	1.5×10^{-13}	MSSM

FIGURES

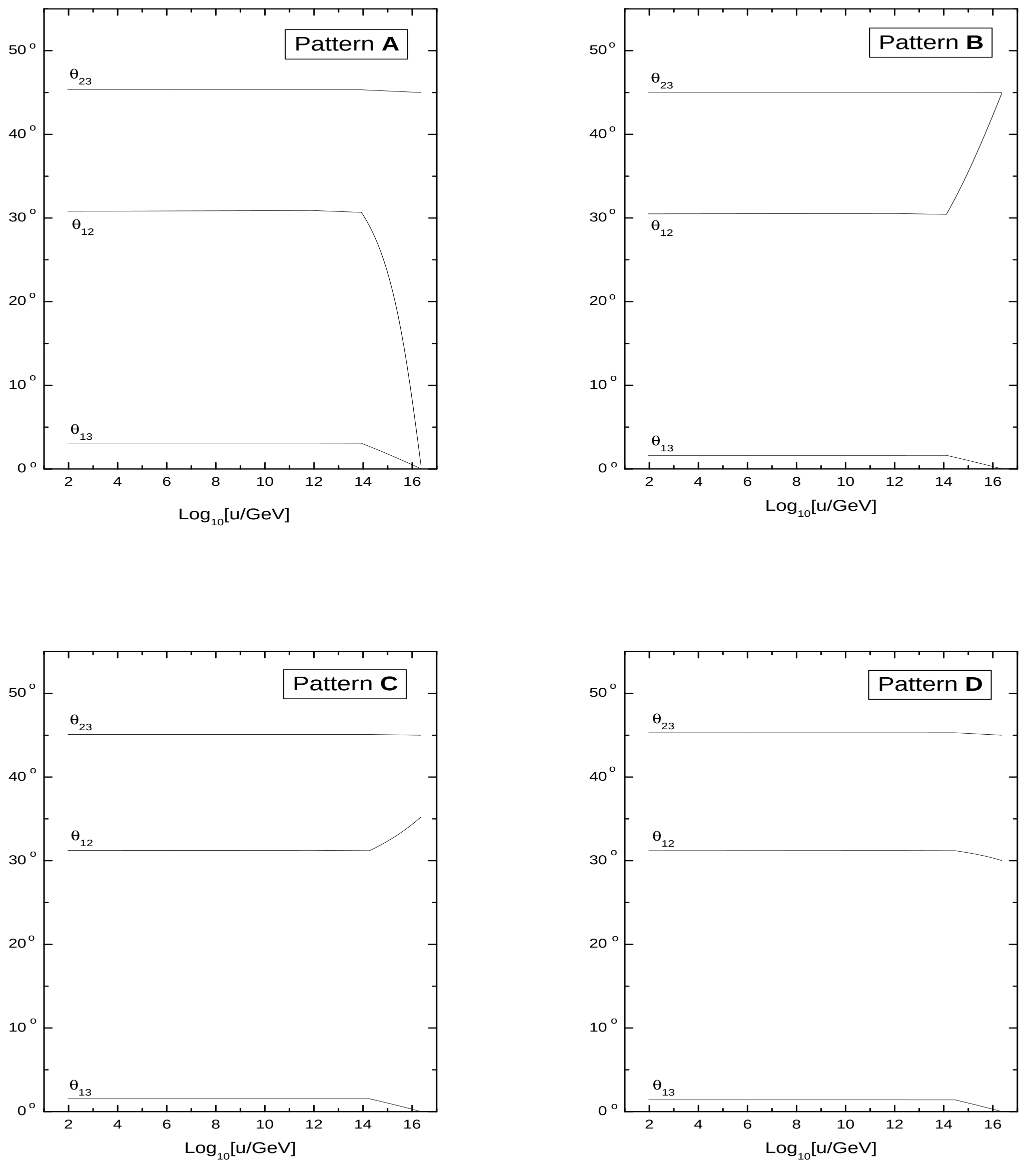

FIG. 1. The RGE evolution of three lepton mixing angles between $\Lambda_{\rm EW}$ and $\Lambda_{\rm GUT}$ in the SM, where the initial values of relevant parameters are listed in Table 1.

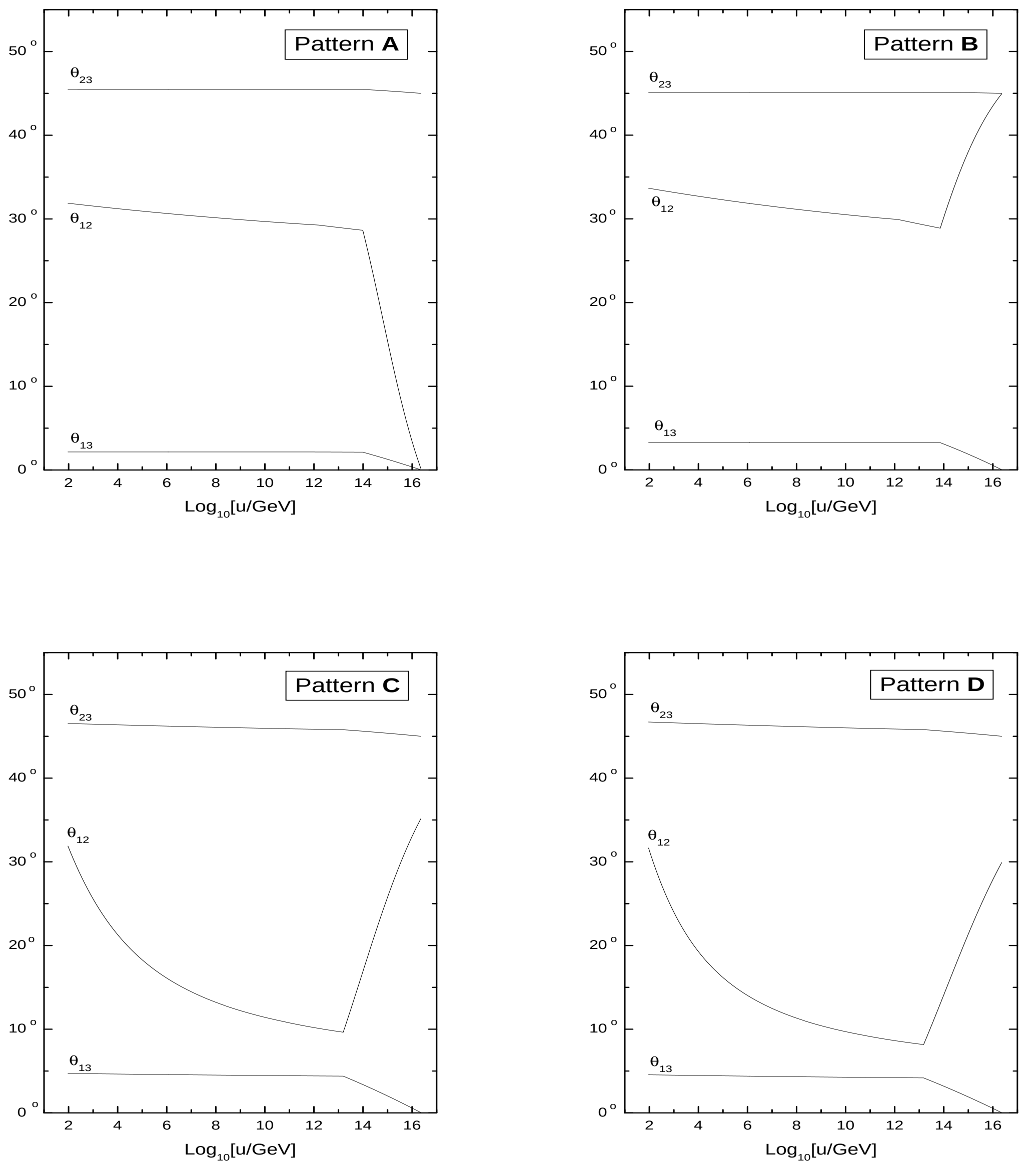

FIG. 2. The RGE evolution of three lepton mixing angles between $\Lambda_{\rm EW}$ and $\Lambda_{\rm GUT}$ in the MSSM, where the initial values of relevant parameters are listed in Table 1.

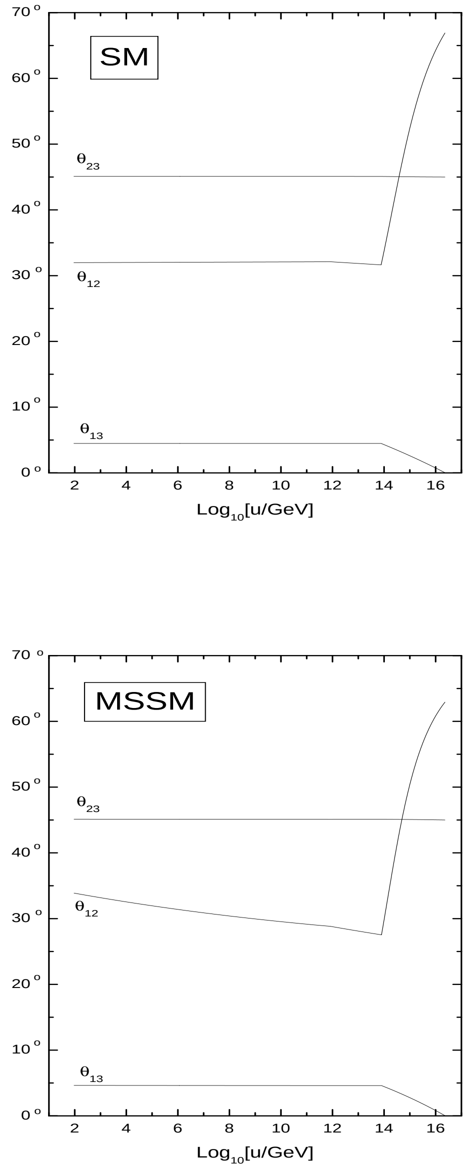

FIG. 3. Running of three lepton mixing angles, where $m_1 = 0.15$ eV, $\Delta m^2_{\rm sun} = 3.5 \times 10^{-4}$ eV^2, $\Delta m^2_{\rm atm} = 7.6 \times 10^{-3}$ eV^2, $y_\nu = 0.9$, $r_1 = 1/41$, $r_2 = 1/17$, $\{\theta_{12}, \theta_{23}, \theta_{13}, \delta\}_{V_{\rm MNS}} = \{67^\circ, 45^\circ, 0^\circ, 94^\circ\}$, $\{\theta_{12}, \theta_{23}, \theta_{13}, \delta\}_{V_\nu} = \{0^\circ, 0^\circ, 45^\circ, 0^\circ\}$ and $m_H = 120$ GeV have typically been input at $\Lambda_{\rm GUT}$ in the SM; and $m_1 = 0.12$ eV, $\Delta m^2_{\rm sun} = 3.9 \times 10^{-4}$ eV^2, $\Delta m^2_{\rm atm} = 5.4 \times 10^{-3}$ eV^2, $y_\nu = 0.8$, $r_1 = 1/41$, $r_2 = 1/15$, $\{\theta_{12}, \theta_{23}, \theta_{13}, \delta\}_{V_{\rm MNS}} = \{63^\circ, 45^\circ, 0^\circ, 100^\circ\}$, $\{\theta_{12}, \theta_{23}, \theta_{13}, \delta\}_{V_\nu} = \{0^\circ, 0^\circ, 45^\circ, 0^\circ\}$ and $\tan\beta = 10$ have typically been input at $\Lambda_{\rm GUT}$ in the MSSM.